Why the Expansion of the Universe is Accelerated

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Connection Gravitation-Thermodynamics 1

Equivalence principle at work:

Tolman's law: $T\sqrt{-g_{tt}} = \text{constant}$ (Tolman, 1934).

Heat flow through an accelerate body (Eckart, 1940).

Gravothermal catastrophe (Antonov, 1962) $\rightarrow$ Newtonian versus Einstein Gravity.

$\Rightarrow$ Entropy cannot become arbitrarily large.

Newtonian gravity is inconsistent with thermodynamics.

Connection Gravitation-Thermodynamics 2

Black holes $\rightarrow$ $T_{bh} = 0$ versus $T_{bh} \propto 1/M$, $S_{bh} \propto \mathcal{A}_{bh}$.
(Bekenstein, 1974, 1975; Hawking, 1975).

Cosmological event horizons possess $S \propto \mathcal{A}_{eh}$, $T_{eh} \propto \kappa$
(Gibbons and Hawking, 1977).

Black holes and cosmological horizons possess entropy and temperature. Both behave thermodynamically in spite of their negative heat capacity.

The second law

Ordinary systems tend to thermodynamic equilibrium

$$S' \geq 0 \qquad \text{and} \qquad S'' \leq 0$$

The Universe's entropy is contributed by the Horizon's entropy, S_H, and the Fluid's entropy, S_f, within the horizon.

$$S_H \propto \mathcal{A}$$

Hubble horizon, $l_H = H^{-1}$

$$\mathcal{A} \propto \frac{1}{H^2} = \frac{3}{2G}\,\frac{1}{\rho}$$

The second law

Therefore,

$$\mathcal{A}' = \frac{9}{2G}\,\frac{1\,+\,w}{a\,\rho} \qquad (w = P/\rho)$$

phantom models, $1\,+\,w < 0$, violate the second law.

$$\mathcal{A}'' = \frac{9}{2G\,a^2\,\rho}\,(1\,+\,w)\,(2\,+\,3w)$$

Accordingly, $\mathcal{A}'' \leq 0$ for $-1 \leq w \leq -2/3$, and $\mathcal{A}'' > 0$ otherwise. Thus,

universes dominated by fluids at late times with constant equation of state either of phantom type or larger than $-2/3$ do not tend to thermodynamical equilibrium.

The second law

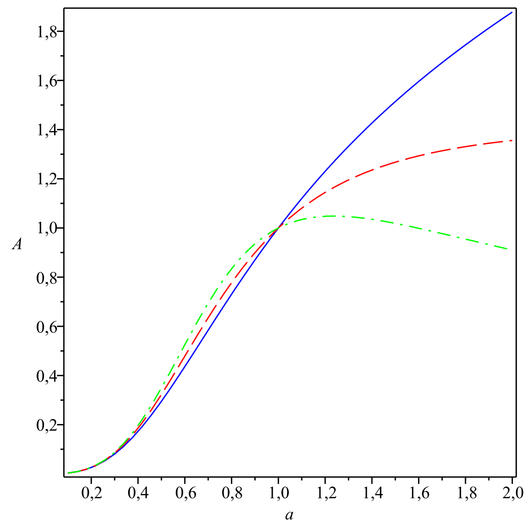

Figure: Evolution of the Hubble horizon area with the scale factor. $w = -5/6$ (solid line), $w = -1$ (dashed), and $w = -1.2$ (dot-dashed).

The second law

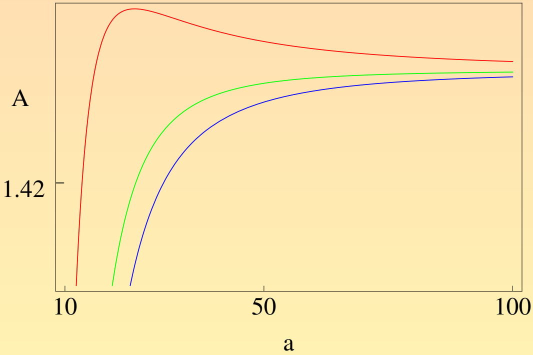

Figure: Evolution of the horizon area with the scale factor for a universe dominated by CDM and cosmological constant, $w = -1$. From top to bottom $\Omega_{k0} = -0.02$, 0, and 0.009, respectively. In all three cases $\Omega_\Lambda = 0.7$.

Incorporating the fluid entropy

What about the fluid entropy? Is the second law really fulfilled?

$$S'_f \, + \, S'_H \geq 0\,?$$

$$\frac{S'_f}{S'_H} \propto a^{3(w-1)/2} \quad \Rightarrow S'_f \, + \, S'_H \geq 0 \quad \text{for } w < 1 \text{ when } \; a \to \infty$$

Likewise, is $S''_f \, + \, S''_H \leq 0$ in the long run?

$$\frac{S''_f}{S''_H} \propto a^{3(w-1)/2} \quad \Rightarrow S''_f + S''_H \leq 0 \quad \text{for } -1 < w < -2/3 \text{ when } \; a \to \infty$$

$w \neq$ constant

Barboza-Alcaniz model (PLB (2008))

$$w(z) = w_0 + w_1 \frac{z(1+z)}{1+z^2} \qquad (1+z=1/a)$$

The accessible range

$$\begin{cases} w_1 < 2/3 \\ -1 < w_0 < -2/3 \end{cases} \cup \begin{cases} 2/3 \leq w_1 < 1 \\ -1 < w_0 < -w_1, \end{cases}$$

comes to be fully consistent with the observational constraints from SNIa, BAO & CMB data

$$-1.35 \leq w_0 \leq -0.86\,, \qquad -0.33 \leq w_1 \leq 0.91$$

Chaplygin gas

$$p = -A/\rho \quad \Rightarrow \; \rho = \sqrt{a + (B/a^6)}$$

The evolution of the Hubble's horizon area, $\mathcal{A} \propto 1/\rho$, is akin to the one depicted by the dashed line in Fig.1.

Moreover,

$$\frac{S'_{Ch}}{S'_H} \to 0 \quad \& \quad \frac{S''_{Ch}}{S''_H} \to 0 \; \text{ when } \; a \to \infty$$

Therefore

$$S'_{Ch} + S'_H > 0 \quad \& \quad S''_{Ch} + S''_H < 0 \quad \text{when} \quad a \to \infty$$

Modified Gravity Models: DGP

The Dvali-Gabdaze-Porrati model (PLB (2000)) presents

$$H^2 + \frac{k}{a^2} = \left(\sqrt{\frac{\rho}{3M_{Pl}^2} + \frac{1}{4r_c^2}} + \frac{1}{2r_c} \right)^2$$

$$S_H = k_B \frac{3\pi \, \tilde{r}_A^2}{\ell_{pl}^2} \left[1 + \frac{\tilde{r}_A}{r_c} \right] \qquad \left(\tilde{r}_A = \frac{1}{\sqrt{H^2 + (k/a^2)}} \right)$$

We found,

$$\begin{aligned} S' &= S'_A \left[1 + \frac{S'_m}{S'_A} + \frac{S'_r}{S'_A} \right] \to 0 \,, \\ S'' &= S''_A \left[1 + \frac{S''_m}{S''_A} + \frac{S''_r}{S''_A} \right] \to 0 \,. \end{aligned}$$

Modified Gravity Models: Cardassian

Spatially flat FRW, dust dominated, model with generalized Friedmann equation (Freese et al., PLB (2002))

$$H^2 = \frac{8\pi G}{3}\,\rho \, + \, B\rho^\alpha$$

At the background level every cardassian model can be mapped onto some dark energy dominated model with $k = 0$ satisfying

$$\alpha = 1 \, + \, w \quad \text{and} \quad B = (8\pi G/3)\,(\rho_{x0}/\rho_{m0}^{\alpha})$$

As a consequence, $S_H'' < 0$ for sensible α values. Also, it can be seen that

$$S_m' \, + \, S_H' > 0\,, \quad \text{and} \quad S_m'' \, + \, S_H'' < 0 \quad \text{as} \quad a \to \infty$$

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- The entropy of the Universe seems to tend to some maximum value (of the order of H^{-2} when $a \to \infty$), but in order to reach a firmer conclusion, accurate measurements regarding $H(z)$ are called for.
- Finally, the existence of dark energy or -equivalently- some modified gravity theory could have been expected on thermodynamic grounds.

Dark matter yielding to dark energy